

Prospective middle school mathematics teachers' knowledge of linear graphs in context of problem-posing

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Abstract

This study examined prospective middle school mathematics teachers' problem-posing skills by investigating their ability to associate linear graphs with daily life situations. Prospective teachers were given linear graphs and asked to pose problems that could potentially be represented by the graphs. Their answers were analyzed in two stages. In the first stage, the problems were evaluated in terms of whether they represented daily life situations or not and in the second stage, the conceptual validity of the responses was examined. Prospective teachers were found to experience difficulties in selecting stories that were appropriate for the structures of the linear graphs and in accurately conveying the data in the graphs through their stories. Of the five types of errors identified in the problems posed, the failure to express linearity was the most common. In addition, statistical analyses showed that success in problem-posing declined as the complexity of the data in the graphs increased.

Keywords: Problem-posing, linear graphs, prospective middle school mathematics teachers.

Introduction

Content knowledge is one type of knowledge that teachers of mathematics need to possess to ensure student achievement (Aslan-Tutak & Adams, 2015; Ball, Thames & Phelps, 2008; Goos, 2013; Mewborn, 2001). This is because mathematics content knowledge is a crucial factor influencing the quality of mathematics teaching (Ball, Lubienski & Mewborn, 2001). According to National Council of Teachers of Mathematics [NCTM] (2000), teachers should have an in-depth understanding of the mathematical concepts they teach. Similarly, Ma (2010) stated that teachers should have a profound understanding of the mathematical concepts they use in their teaching. One of the assessment tools used to examine teachers' mathematical knowledge and to identify their errors and conceptual misunderstandings is problem-posing (Kılıç, 2013; Rizvi, 2004; Ticha & Hospesova, 2009). Stoyanova (1998) emphasizes the common agreement among researchers that the problems posed by students provide important clues about their mathematical skills. The current study therefore examined prospective middle school mathematics teachers' mathematics knowledge, using linear graph problem-posing activities that emphasize the skill for translating between representations.

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Students' ability to understand and use representations is influenced by their teachers' knowledge of representation (Hjalmarson, 2007; Stylianou, 2010). Teachers must possess fluent knowledge about representation types and the transition among these types in order to establish a conceptual learning environment (Ball, Hill & Bass, 2005; McAllister & Beaver, 2012). Problem-posing is an important assessment tool for determining the ability to transition among representation types. Friedlander and Tabach (2001) argue that problem-posing is a frequently used tool when translating from different types of representation to daily life situations. Walkington, Sherman and Howell (2014) found that personalized problems related to students' out-of-school interests are more effective at improving achievement with regard to linear functions. Moreover, these researchers argue that personalization can be accomplished through simple mathematics story problems and that problem-posing can serve as an important tool in this context. In addition, many studies confirm that problem-posing can serve as a means of associating mathematical concepts with daily life situations and can thus contribute to mathematics learning (Abu-Elwan, 2002; Dickerson, 1999; English, 1998). Therefore, the investigation of content knowledge through problem-posing should provide us with an accurate assessment of whether prospective teachers have the skills that their students are expected to acquire.

A fundamental mathematical domain in middle school mathematics involves the concept of functions, particularly functional relations of the form $y=mx+b$ (Brenner et al., 1997; Ministry of National Education (MONE), 2013; NCTM, 2000). According to Wilkie (2014), many real-world applications are modeled as functions and significant emphasis is placed on functional thinking in mathematics courses during the later years of schooling. In addition, functional relationships play a key role in building algebraic thinking. The lines of the graphs used for problem-posing in this study have the form $y=ax+b$. Therefore, problems involving such graphs may provide significant evidence for understanding how prospective teachers perceive visually presented functional relations.

Theoretical Framework

Problem-posing and the Classification of Problem Posing Activities

Problem-posing, also referred to as problem generation or problem finding, is defined as the process of generating new problems or reformulating existing ones (Akay, 2006; Leung, 1993). In his classification of problem types, Pehkonen (1995) includes problem-posing in the category of open problems. Different theoretical frameworks are offered in the literature for classifying problem-posing activities, each using different criteria (e.g., Christou, Mousoulides, Pittalis, Pantazi-Pitta & Sriraman, 2005; Contreras, 2007; Silver, 1995; Stoyanova & Ellerton, 1996). Silver (1995), taking into account the relationship between problem-posing and problem-solving, argues that problem-posing can happen prior to problem-solving, during problem-solving, or after problem-solving. Christou et al. (2005) formulated a classification according to the relationship of the quantitative information of the problem-posing tasks with thinking processes. This classification consists of the following categories: editing quantitative information, selecting quantitative information, comprehending and organizing quantitative information and translating quantitative information.

Stoyanova and Ellerton (1996), on the other hand, classify problem-posing activities as those taking place in free situations, structured situations and semi-structured situations. In free problem-posing, students are asked to generate problems on the basis of a given natural situation, without any limitations (e.g., generating a shopping problem). In semi-structured problem-posing, students are given open-ended situations and are asked to pose problems on the basis of these situations (e.g., problem-posing related to daily life situations to represent the data in a linear graph). The researchers (Abu Elwan, 1999; Christou et al.,

2005; Stoyanova, 1998) indicated that problem-posing activities related to graphs, open-ended stories and a picture or diagram were included in the semi-structured problem-posing activities. In the current study, prospective teachers were expected to pose problems appropriate to linear graphics related to daily life situations. Therefore, these types of problem-posing activities took place in this category. In structured problem-posing, students are asked to generate problems appropriate for a specific solution strategy. This classification is widely used by researchers.

Representations and Transitions between Representations

Representations and transitions between representations play an important role in the teaching of mathematical concepts. Ainsworth (2006) draws attention to their importance by stating that two representations are better than one. Gagatsis and Shiakalli (2004) argue that one representation renders only some aspects of the concept visible, whereas multiple representations complement one another in elucidating a concept. In addition, the ability to translate between different representations has been found to be a strong indicator of conceptual understanding (Lesh, Post & Behr, 1987; Panasuk & Beyranvand, 2010; Stylianou, 2010; You & Quinn, 2010). According to Harries and Suggate (2006), the use of multiple representations provides different views of mathematical concepts, while transitions between representations improve understanding. The importance of using representations can be seen in mathematics curricula. Bal (2014) argues that representations are part of the skill set emphasized in mathematics curricula, which includes communication, association and problem-solving, while NCTM (2000) emphasizes that mathematical knowledge can be developed by the generation, comparison and utilization of representations. In the mathematics curricula of Turkey, it is advised that concepts and rules be taught by using different types of representations that are associated with one another.

Representations are classified into internal and external representations (Goldin & Shteingold, 2001; Harries & Suggate, 2006; Lesh, et al., 1987). Internal representations, as creations of the human mind, are employed to assign meaning to mathematical concepts and operations (Goldin & Shteingold, 2001; Hjalmarson, 2007). External representations, on the other hand, are used to represent objects and things outside of the human mind (Goldin & Shteingold, 2001; Lesh et al., 1987). Mathematical equations, algebraic expressions, graphs and geometric shapes are examples of external representations. External representations are further subdivided into transformations and translations (Lesh et al., 1987). Transformations take place within a single representation, whereas translations take place between two or more representations (Lawler, 2000). For example, if a linear function of the form $ax+by=c$ is represented in different ways, this is called transformation. If a graph is turned into an algebraic representation or an algebraic representation is used to model a real life situation, this is called translation (You, 2006). Hence, a translation will always involve two modes of representation: a given source representation and a specified target representation (Adu-Gyamfi, Bosse & Stiff, 2012).

In this study, the ability to translate between representations was limited to translations between linear graphs and daily life situations. These activities correspond to the translation subgroup of external representations. This limitation was enforced for a number of reasons. First, the visual representation of a functional relationship expresses information much more clearly than an algebraic expression. Knuth (2000) makes the point that graphical representation displays an infinite number of points and presents information very clearly, whereas information is shown indirectly in algebraic representations. The current study aimed to examine prospective teachers' conceptual knowledge in situations where they are faced with a clear presentation of information. Second, it was observed that most studies examining the ability to translate between

representations through problem-posing have focused on problem-posing based on symbolic representations (e.g., Işık & Kar, 2012; Kar, 2015; MacAllister & Beaver, 2012; Toluk-Uçar, 2009). Only a limited number of studies on problem-posing using graphs have included a linear equation in the form of $y=ax+b$ (e.g., Cai et al., 2013). To the best of the author's knowledge, no studies have examined problem-posing skills by using graphs that emphasize a functional relationship and in which two lines intersect with one another. Third, no studies focusing on the errors in problems posed for linear graphs have been found. This study was therefore limited in such a way that it will help to fill these gaps in the literature.

Studies on Problem-posing Involving Linear Graphs

Many studies involving translations between symbolic, tabular and graphical representations have been carried out (e.g., Adu-Gyamfi et al., 2012; Cunningham, 2005; Friedlander & Tabach, 2001). In a study conducted with 43 students attending an algebra course, Adu-Gyamfi et al. (2012) classified the errors made when translating between symbolic, tabular and graphical representations into three groups: implementation errors, interpretation errors and preservation errors. Implementation errors arise from computational mistakes (for example, calculating x as -6 for $2x+3=y$ and $y=15$). In interpretation errors, mathematical concepts are misinterpreted (for example, misinterpreting the slope when formulating the equation representing data in a table). Finally, preservation errors arise when some of the characteristics of the representation are translated correctly and others are not (for example, when drawing a graph for the equation $3y-6x=9$, the steps required for drawing the graph are followed correctly, but the line cuts the x axis at the wrong point). This type of error is usually made towards the end of the translation process. Cunningham (2005) examined the types of transfer problems employed by algebra teachers in their teaching and assessment and found that graphic to numeric transfer problems occurred less frequently than any other type of transfer problem.

However, the number of studies investigating the relation of linear functions and in particular, linear graphs to daily life situations by problem-posing is limited (Cai et al., 2013; Huang & Kulm, 2012; Işık & Kar, 2012; Walkington & Bernacki, 2014; You, 2006; You & Quin, 2010). In some research (e.g., You, 2006; You & Quin, 2010), the prospective teachers' skill at translating between the representations has been investigated using quantitative methods. In such research, the translation from algebraic representations into daily life situations is taken into consideration. For example, You (2006) found that most elementary and middle school prospective teachers (97 out of 104) were able to solve story problems symbolically, but only 18 were able to generate story problems for the equation $y = 6x + 2$. The author also determined that pre-service teachers had difficulty understanding the relationship expressed in a linear function (for example, Kim has six times as many soccer balls as Bob and then Kim got two more). Similarly, Huang and Kulm (2012) found that pre-service teachers were not very successful at creating story problems for given non-linear graphs and that they used visual judgment rather than logical reasoning.

According to Bosse, Adu-Gyamfi and Cheetham (2011), mathematics teachers believe that student achievement in terms of generating verbal stories on the basis of a table or a graph is very low, and they make infrequent use of such activities in their teaching environments. Cai et al. (2013) examined 11th-grade students' problem-posing and problem-solving skills by using a linear graph of the form $y=ax+b$ and a system of equations expressed in algebraic form. The researchers found that most errors were related to expressing the starting point of the graph and the slope. In addition, they found that only 16.6% of the students were able to pose problems about a graph involving an equation of the form $y=ax+b$. Işık and Kar (2012) investigated the errors in the problems posed for equation systems by prospective mathematics teachers and found that most errors fell into the categories pertaining to

incorrect translation of mathematical notations into problem statements, ignorance of the realism of problems, failure to establish a relation between the variables, and a lack of conceptual information about equations. Walkington and Bernacki (2014) examined the difficulties students in grades 6-10 faced when posing and solving problems involving linear equations. They found that students primarily had difficulties using precise language and conceptualizing a functional relationship between unknown quantities. Instead of posing problems defining a general linear relationship, students posed problems supplying a specific value for one quantity and asking the solver to calculate the other (e.g., $\text{cost} = 1.25 \times \text{songs} - 15$).

Background Information: Education System in Turkey and Middle School Mathematics Teaching

The education system in Turkey consists of five hierarchical levels: kindergarten, primary school (grades 1-4), middle school (grades 5-8), high school (grades 9-12) and university. There is a standard curriculum for each of these different levels of education. The middle school mathematics curriculum consists of five learning domains: numbers and operations, algebra, geometry and measurement, data analysis and probability. In each of these learning domains, teachers are advised to use problem-posing and problem-solving together. The curriculum adheres to the belief proffered by Gonzales (1998) that problem-posing is the fifth step of problem-solving. Generating new problems that resemble problems already solved and posing realistic problems involving given situations are also emphasized (MONE, 2013).

Middle school mathematics teachers in Turkey graduate from four-year bachelor's degree programs offered by the education faculties of universities. Entry into these programs is competitive and determined by a national university entrance exam, which is designed to measure 12 years of learning. Students are placed in middle school mathematics teaching programs according to their level of achievement on the entrance exam. Teachers who graduate from the four-year program teach mathematics to students in grades 5-8 in middle school (ages 12-15). The curricula of middle school mathematics teaching programs consist of courses on general education, pedagogy and mathematical content knowledge. During their junior year at university, students are also offered a course on the methods of mathematics teaching. This course introduces the concepts that are to be taught in middle school classes and provides applied training about the activities to be used when teaching these concepts. In their senior year, prospective teachers also participate in actual teaching at schools and are able to observe and perform in-class teaching activities.

Method

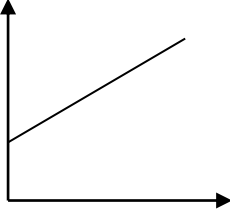
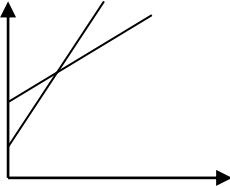
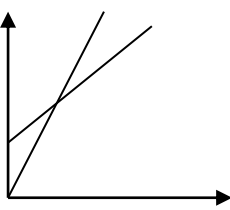
Participants

Prospective teachers in the fourth year of a middle school mathematics teaching program at a university in the eastern part of Turkey were informed about the aim of the present study. At the end of this information stage, 93 prospective teachers agreed to participate in the study on a voluntary basis. Each participant was assigned an identification code by the researcher (PT1, PT2, ...PT93). Prospective teachers who participated in the study had experience with linear graphs from their past schooling, particularly middle and high school. All participants in the study had also received instruction on how to use these graphs in teaching during their third-year course on the methods of mathematics teaching. In this course, they performed problem-posing activities involving linear graphs and discussed the errors they made. In addition, as fourth-year students, they had observed lessons and analyzed the work of students in middle schools.

Data Collection and Analysis

This study used the Problem-posing Test (PPT) to collect data (see Table 1). The PPT was administered to prospective teachers during their class hours and they were given as long as they needed to complete the test. The prospective teachers were asked to pose real life problems that could be represented by the given graphs. All the lines in the graphs were located in the first quadrant of the coordinate plane, but the graphs differed in certain respects. In the first item, there was a single line that started from the positive y axis. In the second and third items, the graphs contained two intersecting lines, but their starting points were different. Both of the lines in the second item started from the positive y axis, whereas in the third item, one of the lines started from the $(0,0)$ point. More graphs could have been included, but the number of items was kept low to allow for in-depth analyses. Graphs included in the PPT were selected from among those most commonly encountered in the curricula and in textbooks.

Table 1. *PPT items and their characteristics*

Items	Characteristics
	<p>Graph starts from the positive y axis.</p>
	<p>Lines start from the positive y axis.</p> <p>The two lines intersect.</p>
	<p>One of the lines starts from the positive y axis and the other starts from point $(0, 0)$.</p> <p>The two lines intersect.</p>

Story problems involving daily life situations posed on the basis of given graphs should express the starting point of the graph, the functional relationship between x and y and linearity. Thus, Cai et al. (2013) argue that these problem-posing activities help to assess conceptual understanding, more so than procedural knowledge. Prior to administering PPT in this study, it was subjected to 20 prospective teachers as a pilot study. It was applied to prospective teachers and then they were interviewed. In the pilot study, the numeric values applied in the graphs were included in the PPT. In the pilot study carried out during development of the PPT, prospective teachers stated that the numeric data on the graphs made it more difficult for them to devise stories. In addition, prospective teachers indicated that they could pose more appropriate problems if they determined the numbers

themselves. Since this study is concerned with participants' conceptual understanding, what matters is not the type of story but the concepts used to describe the situation. There were no numbers on the graphs. In their stories, participants were free to choose any numbers as long as they were consistent with the structure of the graph.

Problems posed on the basis of the graphs given in the PPT were analyzed using a three-point scale: failing (0 points), weak (1 point) and good (2 points). This scale was determined following a careful review of the literature on graphs, equations and problem-posing (e.g., Cai et al., 2013; Işık, Kar, İpek & Işık, 2012; Kar, 2015; Luo, 2009; You, 2006). The failing category contained responses that failed to associate the graph with a daily life situation, or failed to provide an answer at all. Responses in the weak category did associate the graph with a daily life situation, but the contextual structure of the problem being posed did not correspond to the structure of the graph. At this stage, the participants' responses were subjected to content analysis and error categories were created on the basis of previous work in the field (see section 2.3; for an explanation of error categories and sample responses, see section 5.1). If the elements and constructs in the source representation were successfully articulated in the target representation, the translation was considered to have been successful (Adu-Gyamfi et al., 2012). Thus, the good quality category contained conceptually valid problems that associated the graphs with daily life situations, the solution to which involved drawing the given graphs.

Repeated-measures ANOVA was used to test whether prospective teachers' levels of achievement in posing problems varied between the different graphs. The Bonferroni post hoc test was conducted to identify the items that were responsible for the differences observed. Two researchers, who agreed in their evaluations 92% to 100% of the time, analyzed the problems posed by the participants. Scores were then compared and a consensus was reached regarding the assessment of problems previously scored differently.

Results

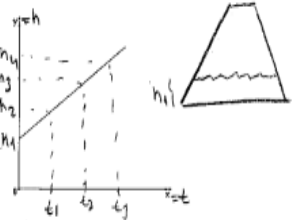
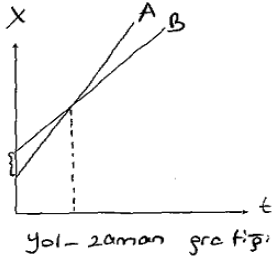
Findings Related to Errors on PPT Items

Responses placed in the *failing* category failed to pose problems involving daily life situations that represented the graph. PT₂, for example, posed the following problem: "Draw a graph for the $y = f(x) = x + 1$ function." PT₂ simply created a function that fit the structure of the graph and directly asked that the graph be drawn. Responses in the *weak* category involved a daily life situation, but the contextual structure of the problem did not correspond to the structure of the graph. Five types of errors (E₁–E₅) were identified in the responses placed in this category: E₁, failure to include linearity; E₂, failure to express the starting point(s) of the line(s); E₃, incompatible story; E₄, failure to include a question root; E₅, logical error. Sample responses for these error types are provided in Table 2.

Responses with the E₁ error failed to construct stories that express linearity. In the first problem in Table 2, the participant is asked about the amount of increase in the height of the water in a bucket being filled with a tap that pours water at a constant rate relative to time. Due to the shape of the bucket, however, the time-height graph that results will not be the same as the graph given in the first item of the PPT. Given that the bucket narrows towards its opening, the increase in the height of the water will be faster over time. In other words, the graph would be parabolic instead of linear. Thus, this problem contains an E₁ error. Responses with the E₂ error failed to express the starting point(s) of the line(s) in their daily life stories. The second problem in Table 2, which was prepared in response to the first item on the PPT, involved a year-height graph, but failed to express the initial height of the tree. Additionally, the expression "grows two meters each year" is not

sufficient to express linearity, because it was not made clear whether the tree had a constant rate of growth over the span of the year. Thus, this problem also represents an E₁ error.

Table 2. Sample responses classified as weak problems

Weak Problems	Type of Error	
<p>1.</p> 	<p>The bucket in the figure is partially filled with water. This bucket is being filled with water at a constant rate. Draw a graph that shows the height of the water in the bucket over time. (PT₂₉)</p>	E ₁
<p>2. There is a tree that grows two meters each year. How tall will this tree be at the end of four years? (PT₅₀)</p>	E ₁ and E ₂	
<p>3. Ahmet and Ali are saving their money. Ali has 10 liras. Ahmet initially has no money. Ali saves 2.5 liras a day, whereas Ahmet saves 5 liras a day. How many days will it take for Ahmet to have more money than Ali? (PT₄₉)</p>	E ₃	
<p>4. Vehicle A starts accelerating from the fifth second onwards and increases its speed. (PT₆₁)</p>	E ₁ and E ₄	
<p>5.</p>  <p>(distance-time graphics)</p>	<p>There are two vehicles, A and B. These vehicles are located 10 and 15 meters away from the starting point. Vehicle A has a velocity of 5m/s and Vehicle B has a velocity of 7 m/s. When will these vehicles meet? (PT₂₂)</p>	E ₅

Responses with the E₃ error failed to express continuity, as required by the graphs. The third problem in Table 2, which was prepared in response to the third item of the PPT, included this mistake. This problem related a story involving an amount of money saved daily. Since days are represented in natural numbers, the graph resulting from this problem will not be the same as the given graph (that is to say, the graph showing Ahmet's total savings, for example, will consist of pairs in the form of (0, 0), (1, 2.5), (2, 5), ...). Responses with the E₄ error failed to include a question root. The fourth problem in Table 2, prepared in response to the first item of the PPT concerning the acceleration of a vehicle, did not include a question root. Moreover, it was not made clear how the acceleration proceeded. Thus, it was not certain that the resulting graph would be a linear one and the problem therefore also contains an E₁ error. Responses with the E₅ error contained stories that were logically inconsistent with the graphs given. The fifth problem in Table 2, prepared in response to the second item of the PPT, involved a road-time graph. The story stated that

there was a five-meter gap between the two vehicles. Since Vehicle A moves at a higher speed than Vehicle B, it is clear that the gap between the two vehicles will become larger over time and the resulting lines will not intersect. Thus, this problem contains an E₅ error.

Prospective Teachers' Achievement in Posing Problems by Graph Type

Table 3 reports the distribution of the failing, weak and good problems posed by prospective teachers for each of the three linear graphs on the PPT.

Table 3. *Distribution of scores for problems posed by prospective teachers*

	Failing	Weak	Good	Mean Score	Total
Item 1	4 (4.3)	56 (60.2)	33 (35.5)	1.31	93 (100)
Item 2	10 (10.8)	64 (68.8)	19 (20.4)	1.1	93 (100)
Item 3	12 (12.9)	60 (64.5)	21 (22.6)	1.1	93 (100)

* Data presented in the form of frequencies (percentages).

Table 3 shows that less than 36% of the problems posed for each of the items included in the PPT were categorized as good. For the second and third graphs on the PPT, achievement levels were even lower. These findings indicate that the prospective teachers had low levels of success in posing conceptually valid problems. When mean scores were compared, it could be seen that participants were more successful for the first item compared with the second and third items. Thus, the inclusion of two lines instead of one resulted in lower levels of achievement.

Results of the repeated-measures ANOVA showed that the prospective teachers' levels of achievement varied significantly by graph type [$F(5.297, 173.653)=3.20, p=.007$]. Results of the Bonferroni post hoc test showed that there were significant differences between the scores for the first and second items ($p=.008<.05$) and between the scores for the first and third items ($p=.039<.05$). The difference between the scores for the second and the third items, on the other hand, was not statistically significant ($p=1.00>.05$). These findings indicate that prospective teachers had greater difficulty posing problems for the second and third graphs than for the first graph. These findings also indicate that the variation in the starting points of the graphs in the second and third items (containing two lines each) did not have a statistically significant impact on problem-posing achievement.

More than 60% of the problems posed were categorized as weak for each of the items included in the PPT. These findings indicate that the prospective teachers were successful in associating graphs with daily life situations, but had difficulties reproducing the conceptual structure of the graphs in their stories. Table 4 reports the distribution of the five types of errors identified in weak responses by graph type.

Table 4. *Distribution of error types identified in weak problems*

Weak responses	E ₁	E ₂	E ₃	E ₄	E ₅	Total
Item 1 (56 responses)	52(68.4)	13(17.1)	10(13.2)	1(1.3)	-	76 (100)
Item 2 (64 responses)	53(68.8)	8(10.4)	3(3.9)	4(5.2)	9(11.7)	77 (100)
Item 3 (60 responses)	44(55.7)	14(17.7)	9(11.4)	5(6.3)	7(8.9)	79 (100)
Total (180 responses)	149(64.2)	35(15.1)	22(9.5)	10(4.3)	16(6.9)	232 (100)

* Data presented in the form of frequencies (percentages).

Table 4 shows that the 180 responses in the weak category contained 232 errors. On average, there were 1.29 errors per response. The table also shows that E₁ and E₂ errors

were the most common, and that E_4 was the least common. These findings therefore indicate that failure to express linearity and failure to express the starting point(s) of the line(s) were the most common errors made when posing problems involving linear graphs. The findings also show that the failure to include a question root in the problem was the least common error.

Discussion and Conclusion

Many studies and teaching documents support the inclusion of multiple assessment tools in the process of learning (MONE, 2013; NCTM, 2000; Van de Walle, Karp & Bay-Williams, 2009). The fact that individual assessment tools suffer from different problems of validity and reliability renders the use of a variety of combined assessment tools essential. The use of different assessment tools is also necessary to create a broader outlook on mathematical understanding (Hiebert & Carpenter, 1992). In this context, problem-posing has been increasingly emphasized in recent years as an alternative assessment tool (Lavy & Shriki, 2007; Silver & Cai, 2005; Ticha & Hospesova, 2009). Problem-posing gives teachers a good idea of their students' skills, attitudes and conceptual learning (Silver & Cai, 2005). Problem-posing can also be used to assess both teachers' and prospective teachers' grasp of mathematics subjects, as well as that of students (Işık & Kar, 2012; Kılıç, 2013; McAllister & Beaver, 2012). This research utilized problem-posing for assessment purposes in order to investigate prospective teachers' mathematical understanding of linear graphs.

The low percentage of responses categorized as failing for each graph on the PPT indicates that most prospective teachers at least make an attempt to associate graphs with daily life situations. Previous studies have shown that fewer students than prospective teachers make this attempt. For example, Cai et al. (2013) found that only 63.3% of 11th graders made an attempt to pose a problem for the graph task, while Işık et al. (2012) found that most prospective elementary teachers (varying between 87.5% and 100% for different items) made an attempt to associate graphs with daily life situations. Considered together, these results indicate that the level of knowledge and experience possessed by the problem-poser has a significant impact on success in problem-posing.

Prospective teachers generally had difficulties using daily life stories to express the data in the graphs (more than 60% of responses for each item were categorized as weak). The most common error made by the participants in this category was the failure to express linearity. This type of error arises when there is a failure to include statements expressing the linear relationship between x and y . This is also the most common type of error made in research carried out with both students and prospective teachers (e.g., Işık et al., 2012; Walkington & Bernacki, 2014; You, 2006). For example, You (2006) found that prospective teachers tended to construct stories that involved a single pair of values (x, y) instead of defining a general linear relationship. In the present study, however, most participants were able to express a general relationship between x and y (e.g., the car accelerates over time), but failed to express that the slope was constant (e.g., there was no statement expressing that the change in the speed of the car was constant). In addition, it was found that some participants constructed stories involving discrete quantities (see the third sample problem in Table 2). This type of error indicates that the participants were exclusively concerned with posing problems reproducing the overall shape of the graph. Other studies (Berg & Philips, 1994; Capraro, Kulm & Capraro, 2005) have also found that students make the error of drawing discrete graphs for continuous data or continuous graphs for discrete data.

The translation of mathematical expressions into verbal statements requires strong linguistic skills. Thus, a lack of sufficient linguistic skills may be another reason for prospective teachers' low levels of achievement. Many studies (Capraro & Joffrion, 2006;

MacGregor & Stacey, 1993) show that participants have difficulty explaining the relationship between mathematical notations and verbal expressions in story problems. Clement (1982) found that students sometimes assign meanings to the variables in an algebraic expression that are inconsistent with verbal expression (some students, for example, wrote the equation $6y=x$ to represent a verbal expression that in fact denoted $6x=y$). Similar issues were encountered in this study. Data swapped places in the translation from graph to verbal expression, leading to failure in terms of representing the graph accurately. This may have resulted from a failure to understand the graph, or it might be an indicator of linguistic problems.

Lack of knowledge and experience regarding problem-posing result in low levels of success (Kar, 2015; Luo, 2009). Prospective teachers' lack of experience with problem-posing involving linear graphs was likely another reason for the lack of success observed in this study. Participants in the study did have some experience with problem-posing activities from various courses, but their levels of achievement were nevertheless low. This indicates that the development of problem-posing skills, similar to the development of problem-solving skills, takes time. Another finding of the study was that participants were more successful in posing problems for the first item of the PPT and had similarly low levels of achievement for items two and three. The first item of the PPT included a single line, whereas items two and three included two intersecting lines and thus, contained a larger amount of mathematical data compared to the first item. From these observations, we can conclude that as the number of associated variables on a graph increases, success in posing appropriate problems for the graph declines. Other studies on translations between representations have reached similar conclusions. For example, Adu-Gyamfi et al. (2012) found that translations involving representations with larger amounts of data resulted in more errors being made.

A teacher's knowledge has an important effect on student achievement (Ball et al., 2008; Hill, Rowan & Ball, 2005; Kulm, 2008). In other words, if teachers do not have the necessary knowledge and experience regarding the concepts they are supposed to teach, they will have difficulty teaching these concepts. The findings of this study show that prospective teachers have low levels of success in posing problems involving linear graphs and that their efforts were hampered by five types of errors. The distribution of the errors indicates the most important deficiency as a lack of conceptual knowledge, which will also likely be reflected in the problem-posing activities these teachers will organize in future. The findings of this study do not provide conclusive evidence about whether a lack of conceptual knowledge, linguistic problems, or a lack of experience with problem-posing led to the different types of errors. Thus, it is recommended that future studies supplement these findings with interviews, so that the causes of the errors can be probed further. Levels of success did not differ significantly between some of the items on the PPT. Similar additional studies conducted with linear graphs that have different structures will make it possible to enhance understanding of this field. The sample size of the study does not allow for generalization of the conclusions to all prospective teachers in Turkey, a country with low levels of achievement in international comparisons. Similar studies with larger sample sizes should be conducted and results should be compared with results obtained in countries with higher levels of achievement in international comparisons.

The literature fails to provide a sufficient explanation for the difficulties faced by students in translating between representations (Adu-Gyamfi et al., 2012). This study aimed to contribute to the literature by examining the errors made by prospective teachers when translating linear graphs into daily life situations. Errors can be viewed as opportunities for improving learning; moreover, the errors determined in this study can provide guidance to teachers and researchers when designing learning environments involving problem-posing.

Furthermore, problem-posing has recently received increased attention, but studies indicate that problem-posing is still only an emerging topic in mathematics education research (Kitchings, 2014). According to Cai et al. (2013), practitioners are interested in making problem-posing a more prominent feature of classroom instruction. The findings of this study contribute to this effort by providing information on the use of problem-posing for the purpose of translating between representations.



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